

Internet-Based CAD Tool for Design of Gripper Jaws¹

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ABSTRACT

We describe an Internet-based CAD tool that automatically designs gripper jaws to align a given part from an initial orientation to a desired final orientation. We describe algorithmic improvements in computational efficiency, frictional analysis and form closure analysis. We describe the interface and give examples. We then apply the design tool to identify conditions under which jaw designs exist, leading to an analytic condition for parts that cannot be aligned. The CAD tool is online at www.ieor.berkeley.edu/~goldberg/sa-gripper/.

1. INTRODUCTION

Geometric algorithms for design, simulation, and manufacturing are often available only to a small number of researchers and engineers. The Internet makes it possible for public to share resources. Interactive design is essential because we often need to check if the part designed is easy to produce or if we have proper tools to handle it. CAD software with public accessibility can provide rapid feedback to designers.

In this paper, we describe a Java-based CAD software to design robot gripper jaws. As illustrated in Figure 1, users can draw a part, define the COM and range of friction coefficients, and choose the initial and the final orientations; the applet computes, ranks, and displays the resulting jaw designs. Online CAD tools such as this can allow designers to rapidly check manufacturability and modify designs to avoid handling problems on the assembly line.

2. RELATED WORK

Computer-Aided Design for manufacture is an active research topic. Since the introduction of Java in 1996, a number of geometric algorithms have been made available online. The Geometry Center at University of Minnesota maintains an extensive collection of interactive geometric algorithms at [23]. Interactive geometric software is also available at [24] to find Delaunay triangulation using 3D Convex Hulls. The Berkeley Remote Interactive Optimization Testbed (*RIOT*) includes several online optimization algorithms [25].

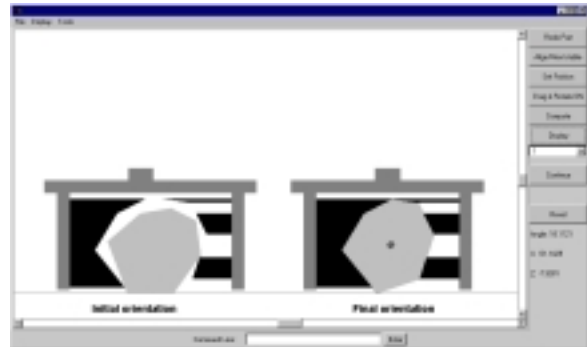


Figure 1 Screenshot of Java applet that automatically designs trapezoidal jaw modules (in black). Jaws will align the user's part from an initial resting orientation to a desired final orientation as the gripper is closed.

Several researchers have developed online CAD software for manufacturing. Wright *et al.* [14][5] develop *Cybercut* [27]: a network CAD/CAM service for design and fabrication of mechanical parts on the Internet. Piccinocchi *et al.* [11] present a Java applet to compare different nonholonomic motion planning algorithms. Goldberg and colleagues [4][18][2] provide an online fixture design system that finds all modular fixtures for a given part consisting three locators and one clamp. Cheng *et al.* [17][6] describe online CAD tools for cam and planar mechanical system design at [26]. Berretty *et al.* [3] describe an online CAD tool for design of traps for the tracks of vibratory bowl feeders. Smith *et al.* [15] develop a Java applet to compute and rank all pairs of parallel-jaw grip points for polygonal parts.

The CAD tool in this paper computes a class of gripper jaws based on rapidly-machineable trapezoidal modules. An optimal design is an arrangement of trapezoidal jaw modules that maximizes contacts between the gripper and the part at its desired final orientation in a form-closure grasp.

Several authors address grasping in the vertical (gravitational) plane. Trinkle and Paul [16] show how to align parts in the gravitational plane by lifting them off work-surface using a planar gripper with two pivoting jaws. Abell and Erdmann [1] study how a planar polygon can be rotated while stably supported by two frictionless contacts. Rao *et al.* [13] give a planar analysis for picking up polyhedral parts using 2 hard point contacts with a pivoting bearing, allowing the part to pivot under gravity to rotate into a new configuration. Lynch [7] [8] derives sufficient

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mechanical conditions for toppling parts in term of constraints on contact friction, location, and motion and we built on his analysis. Zhang *et. al* [19] introduce geometric functions to represent the mechanics of toppling and develop several algorithms for gripper design in [20,22].

3. ALGORITHM

We assume the part can be treated as a rigid extrusion of a polygon, both the part and the jaws are rigid, part geometry, location of the COM and the jaws are known, and that part motion is sufficiently slow to apply quasi-static analysis.

We consider the following design problem. The input is: the vertices of an n -sided convex projection of an extruded polygonal part, its center of mass (COM), initial and desired orientations, a vertex clearance radius ε , and bounds on the part-surface friction coefficient $[0, \mu_{s_max}]$ and on the part-gripper friction coefficient: $[\mu_{t_min}, \mu_{t_max}]$. The output is a report that no solution can be found or a gripper jaw design, specified by an arrangement of trapezoidal jaw modules that is able to topple the part to its final orientation and grasp it in form closure (see Figure 1).

Each jaw module is defined by an *accessible segment* that is the portion of the module that makes contact with the part at its desired orientation. Our objective is to find a set of accessible segments with maximum total length.

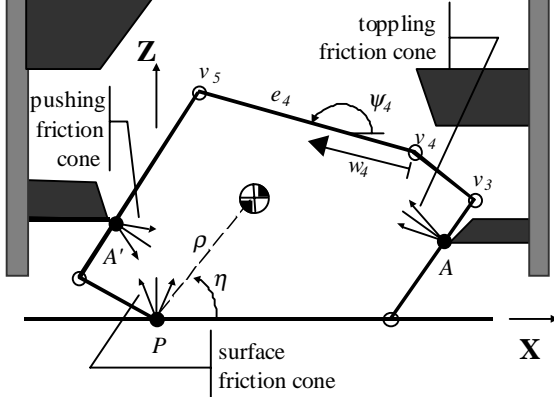


Figure 2 A part in initial orientation with gripper jaws (trapezoidal jaw modules in black).

As shown in Figure 2, the part sits on a flat work-surface at a stable initial orientation. Without loss of generality, we assume the part rotates counterclockwise. We define the World frame, W , to be a Cartesian coordinate system originating at pivot point P with X -axis on the surface pointing right, Z -axis vertical to the surface pointing up. The COM is a distance ρ from the origin and angle η from the $+X$ direction.

The gripper jaws make contact with the part at two points initially. The *pushing tip* A' contacts edge e_i to the left of pivot point P at a distance $z_{A'}$ from the

surface; the *toppling tip* A contacts edge e_j to the right of P at a distance z_A from the surface. Starting from the pivot, we consider each edge of the part in counter-clockwise order, namely e_1, e_2, \dots, e_n . An edge e_k , with vertices v_k at (x_k, z_k) and v_{k+1} at (x_{k+1}, z_{k+1}) , is in direction ψ_k from the X -axis.

Let F_t, F_p , and F_s denote the contact force at A, A' , and the work-surface, respectively, and f_t, f_p , and f_s denote the direction of the corresponding contact force. We also denote the direction at the left edge of the toppling ($/$ pushing $/$ surface) friction cone as f_{tl} ($/f_{pl} / f_{sl}$) and the right edge as f_{tr} ($/f_{pr} / f_{sr}$). Let f_g denote the vertical line through the part's COM. Let $I_{a,b}$ denote the intersection of line a and line b , where a and b can be anyone of these directions.

Let θ denote the rotation angle of the part; initially $\theta = 0$ and finally $\theta = \theta_d$. Let θ_t denote the rotation angle where the COM is right above P .

In [21], we gave an $O(n^5)$ algorithm to find the optimal jaw design; in this paper we improve the algorithm to handle bounded friction coefficients and faster form closure analysis. Here we report details on the resulting $O(n^3 \log n)$ algorithm and its implementation in an online Java applet.

3.1 Bounded Friction Coefficients

In [21] we assumed exact friction values were given. In this section we relax that assumption and derive the toppling conditions given upper and lower bounds on the friction coefficients.

The pushing condition to prevent jamming does not depend on the friction coefficients. It is easy to see that we only need to consider the upper bounds of μ_t and μ_s , i.e. $\mu_s^* = \mu_{s_max}$ and $\mu_t^* = \mu_{t_max}$, to satisfy the pushing conditions to prevent rolling and jamming (see [22] for details).

The *rolling function* and *jamming function* describe the mechanics of toppling. Given $z_{A'}$, the toppling tip makes contact with edge e_j , and the range of friction coefficients, the rolling function $H_j(\theta)$ is the minimum height of A guaranteed to cause the part to rotate at orientation θ and the jamming function $J_j(\theta)$ gives the minimum height of A that is guaranteed not to cause jamming. $H_j(\theta)$ is defined in the range $[0, \theta_t]$.

For each value of θ , the geometric functions span a range of values depending on the range of friction coefficients. To guarantee toppling for all the friction values in the range, we take the maximum values of $H_j(\theta)$ or $J_j(\theta)$ over the given bounds. At each rotational angle θ , the maximum $H_j(\theta)$ or $J_j(\theta)$ corresponds to the *critical friction coefficients* denoted by μ_s^* and μ_t^* . We first consider the functions at each rotational angle θ to derive μ_s^* ; then we find μ_t^* for given μ_s^* and θ .

We consider the case where the COM is to the right of P and A' , i.e., $\theta < \theta_t$ and $x_{A'} < 0$.

Following Mason [9], we construct triangle $P_0P_1P_2$ as shown in Figure 2 if $\gamma_{il} > 2\pi - (\frac{\pi}{2} - \alpha_s) + \theta$, i.e., $\psi_i > \pi + \alpha_s - \alpha_r$, where $P_0(x_{p0}, z_{p0})$ is $I_{sl,pl}$, $P_1(x_{p1}, z_{p1})$ is $I_{sl,g}$, and $P_2(x_{p2}, z_{p2})$ is $I_{pl,g}$. The closed polygon defined by these critical points is called the *critical polygon*, which in Figure 3 is triangle $P_0P_1P_2$. If all the forces within the toppling friction cone generate clockwise moments about the critical polygon, the part will be rotated instantaneously.

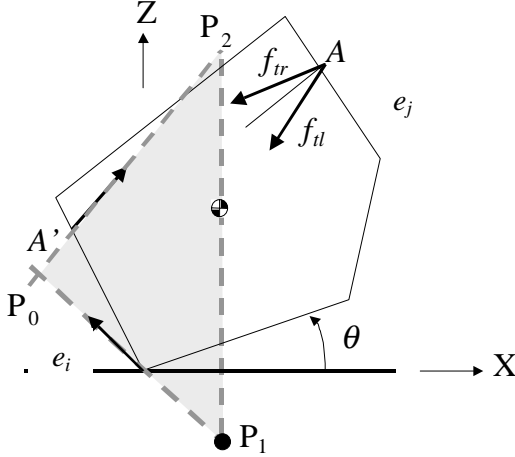


Figure 3 Rolling condition.

As a conservative solution, we find the contact height sufficient to roll the edge by projecting lines from P_0 , P_1 , and P_2 at the angle of f_{il} until they intersect the edge of the part. The intersection with the maximum height of those three is the minimum height sufficient to roll the part.

To relax the exact friction requirement, we note that as μ_s decreases, P_0 moves up along r_{o2} and $H_f(\theta)$ will not increment. It is sufficient to consider only the upper bounds of μ_s , i.e. $\mu_s^* = \mu_{s,max}$, to get $H_f(\theta)$.

Given μ_s^* and θ , the rolling function is a function of μ_t , i.e., $H_f(\mu_t)$. As μ_t decreases, P_2 moves down along r_{l2} but P_0 moves up along r_{o1} . We sample μ_t in the given range, compute the rolling function for each μ_t , and then find μ_t^* for $H_f(\theta)$ using numerical search.

3.2 Form Closure Analysis

We require that the resulting set of the accessible segments generates a form-closure grasp on the part.

We reorder the endpoints of the corresponding accessible segments in counter-clockwise. Starting from the one closest to P , we rename the endpoints $(x_1, z_1), (x_2, z_2), \dots, (x_m, z_m)$. Let V_k denote the unit normal inward vector at contact (x_k, z_k) , V_{kx} (V_{kz}) denote the X (Z)-axis projection of the unit normal inward vector at contact (x_k, z_k) , and T_k denote the torque of V_k respective to P . The *Wrench matrix* $\mathbf{W} =$

$$\begin{bmatrix} V_{1x} & V_{2x} & V_{3x} & \dots & V_{mx} \\ V_{1z} & V_{2z} & V_{3z} & \dots & V_{mz} \\ T_1 & T_2 & T_3 & \dots & T_m \end{bmatrix}. \text{ The Wrench space is}$$

a 3-dimensional manifold defined by V_x , V_z , and T . Each column of \mathbf{W} represents a point in the wrench space. Mishra *et. al* [10] prove that a form-closure grasp is guaranteed if the origin of the wrench space lies strictly in the convex hull of these points.

The algorithm checks the form-closure of the accessible segment set:

1. Compute wrench matrix \mathbf{W} ;
2. Define the set of points, \mathbf{Q} , in wrench space by each column of \mathbf{W} and the origin of wrench space \mathbf{O} ;
3. Find the convex hull of \mathbf{Q} , $\mathbf{CH}(\mathbf{Q})$;
4. If $\mathbf{O} \notin \mathbf{CH}(\mathbf{Q})$, the set of the accessible segments achieve form-closure on the part.

Since it takes time $O(m \log m)$ to construct the 3D convex hull of m points [12] and $m \leq 2n$, the running time to check the form-closure of the set of accessible segments is $O(n \log n)$. Thus, the total running time of the improved algorithm is $O(n^3 \log n)$, an improvement over the $O(n^5)$ of the previous algorithm.

4. IMPLEMENTATION

We have implemented this algorithm into an online Java-based CAD tool. Users can draw a convex polygonal part (up to 10 sides) with a mouse, drag it from the initial resting pose to the desired orientation, and view the resulting designs (if any exist) ranked by their total contact area.

As illustrated in Figure 6, there are three numbers in the right lower corner of the applet: *Angle* is the degree of the part's rotational angle; (X, Z) is the location of your cursor. To use the applet, start by clicking *Design Part* button, then use your mouse to draw a convex polygon above the black horizontal line (counterclockwise). The polygon represents the part and the horizontal line represents the work-surface. You can also drag the part's Center of Mass around using your mouse. The default value of friction cone half-angles is 10 degree. Next, click *Next Stable Pose*, the part will align with the work-surface in one of its stable orientation. Click the button again, the part will rotate to the next stable orientation. Then, you may click *Drag to Desired Pose OFF*. The button will change to *Drag to Desired Pose ON* and a gray image of the part in the initial orientation will display. Now, you can drag the part to rotate it to the desired orientation and *Angle* shows the current rotational angle. We can run the jaw design algorithm by clicking *Design Jaws*. An optimal jaw design will appear. The algorithm requires 7~10 seconds to find and rank jaw designs on a 266MHz Pentium II PC. Black trapezoids represent jaw modules. Move your

cursor close to a vertex of any jaw module, *Angle* disappears and the (X, Z) location of the vertex displays.

To study the sensitivity of the input parameters, we ran the Java applet with varying friction coefficients, part geometry, center of mass, and desired orientation. With friction coefficients of 10°, Table 1 shows simulation results of triangular parts and Table 2 shows those of some more complicated parts. The jaw modules are in black and the arrows indicate the pushing tip and the toppling tip.

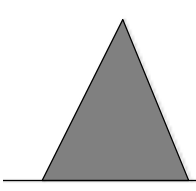

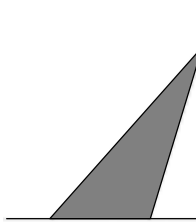

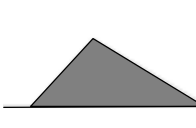

Initial Orientation of Parts	Final Orientation with Optimal Jaw Design	Running Time (s)
		9
		9
		9

Table 1 Simulation results: triangular parts.

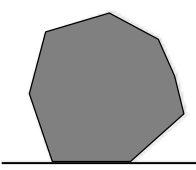
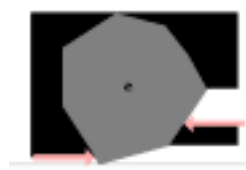
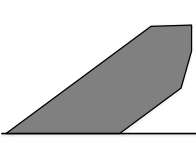

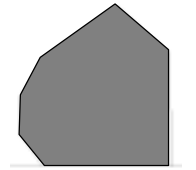

Initial Orientation of Parts	Final Orientation with Optimal Jaw Design	Running Time (s)
		9
		8
		10

Table 2 Simulation results: complicated parts.

5. EXISTENCE OF SOLUTIONS

Some parts, like the last case in Table 1, have no jaw design solution. To characterize such parts, we consider the class of triangular parts shown in Figure 4. Without loss of generality, we assume that edge e_1 is of unit length from (0,0) to (1,0). We assume the part has uniform density.

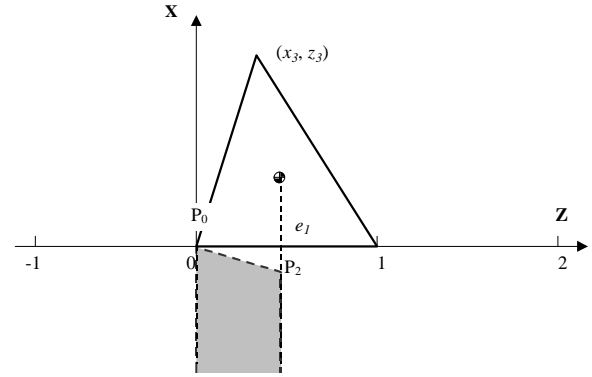


Figure 4 A triangular part.

We are interested in the location of v_3 at (x_3, z_3) that makes the triangular part impossible to topple. Note that $x_3 \in [-1, 2]$ to keep the part stable and e_1 on the work-surface.

If there is no jaw design can initialize the toppling process, it is guaranteed that the part cannot be toppled to its desired orientation. So we consider the initial orientation of the part, i.e., $\theta = 0$.

As shown in Figure 3, that lower A' and smaller the friction coefficients will lead to larger feasible contact area for A . Therefore we consider the extreme condition where the pushing tip A' makes contact with v_1 at (0, 0) with zero friction, i.e., $\mu_l = 0$, $\mu_s = 0$, and $\varepsilon = 0$. By geometric relationship, we have:

$$\tan(\pi - \omega) = z_3 / (1 - x_3) \Rightarrow \omega = \pi - \text{atan}(z_3 / (1 - x_3)),$$

$$\tan(\psi - \pi) = z_3 / x_3 \Rightarrow \psi = \pi + \text{atan}(z_3 / x_3),$$

$$\tan \eta = \frac{z_3 / 3}{(1 + x_3) / 3} \Rightarrow \eta = \text{atan}(z_3 / (1 + x_3)),$$

$$\rho = \sqrt{\left(\frac{1 + x_3}{3}\right)^2 + \left(\frac{z_3}{3}\right)^2}.$$

In this case, the critical polygon is in gray as shown in Figure 4. We find the height sufficient to roll the edge by projecting lines from P_0 and P_2 at the angle vertical to edge e_2 until they intersect the edge. The intersection with the maximum height of those two is the minimum height sufficient to roll the part. The height of the intersection corresponding to P_0 can be shown to be $\frac{z_3(1 - x_3)}{x_3^2 - 2x_3 + z_3^2 + 1}$; the corresponding P_2

can be shown to be $\frac{z_3(1 - x_3)}{x_3^2 - 2x_3 + z_3^2 + 1}$ -

$\frac{z_3 |1+x_3|}{3(x_3^2 - 2x_3 + z_3^2 + 1)}$ (see [22] for details). Thus, the

rolling function $H = \max \left(\frac{z_3(1-x_3)}{x_3^2 - 2x_3 + z_3^2 + 1}, \frac{z_3 |1+x_3|}{3(x_3^2 - 2x_3 + z_3^2 + 1)} \right)$. If $H > z_3$, there is

no jaw design will be able to initialize the toppling process. There will be no jaw design solution if H is higher than $h = z_3$. As shown in Figure 5, a triangular part will have no jaw solution if its vertex v_3 is in the critical area where the rolling function H protrudes from the plane $h = z_3$.

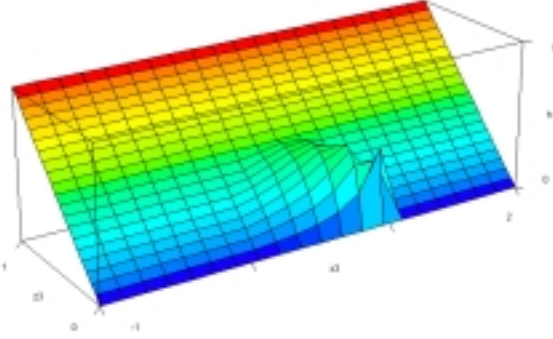


Figure 5 Rolling function H and the plane of $h = z_3$.

No jaw design exists if v_3 is within the boundary of the critical area. The boundary is represented by the intersection of H and z_3 . We consider each term in H , and want to prove that H is determined by $\frac{z_3(1-x_3)}{x_3^2 - 2x_3 + z_3^2 + 1}$. That is equivalent to the following inequality:

$$\frac{z_3(1-x_3)}{x_3^2 - 2x_3 + z_3^2 + 1} \geq \frac{z_3(1-x_3)}{x_3^2 - 2x_3 + z_3^2 + 1} - \frac{z_3 |1+x_3|}{3(x_3^2 - 2x_3 + z_3^2 + 1)}$$

Proof:

$$(x_3 - 1)^2 + z_3^2 \geq 0 \text{ and } z_3 \geq 0$$

$$\Rightarrow x_3^2 - 2x_3 + z_3^2 + 1 \geq 0 \text{ and } z_3 \geq 0$$

$$\Rightarrow 0 \geq - \frac{z_3 |1+x_3|}{3(x_3^2 - 2x_3 + z_3^2 + 1)}$$

$$\Rightarrow \frac{z_3(1-x_3)}{x_3^2 - 2x_3 + z_3^2 + 1} \geq \frac{z_3(1-x_3)}{x_3^2 - 2x_3 + z_3^2 + 1} - \frac{z_3 |1+x_3|}{3(x_3^2 - 2x_3 + z_3^2 + 1)}$$

So the boundary can be determined by:

$$\max \left(\frac{z_3(1-x_3)}{x_3^2 - 2x_3 + z_3^2 + 1}, \frac{z_3 |1+x_3|}{3(x_3^2 - 2x_3 + z_3^2 + 1)} \right)$$

$$- \frac{z_3 |1+x_3|}{3(x_3^2 - 2x_3 + z_3^2 + 1)}, z_3) = z_3$$

$$\Rightarrow \frac{z_3(1-x_3)}{x_3^2 - 2x_3 + z_3^2 + 1} = z_3$$

$$\Rightarrow x_3^2 - x_3 + z_3^2 = 0$$

$$\Rightarrow (x_3 - 0.5)^2 + z_3^2 = 0.5^2$$

Since z_3 is a nonnegative number, the above equation indicates a half-circle as shown in Figure 6. This result implies that any triangular part with the interior angle at v_3 greater 90 degree will be certainly unable to find a jaw design to topple the part.

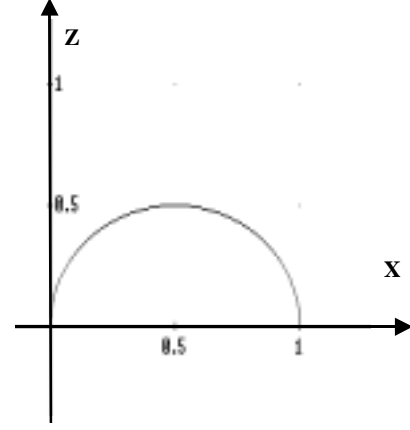


Figure 6 Boundary of v_3 to guarantee no jaw design solution.

We can also interpret this result by geometric construction as shown in Figure 7. E is the intersection of the half-circle and the toppling edge in the initial orientation. In case (a), v_3 is on the circle. The contact force at v_3 passes exactly through one vertex of the critical polygon in gray and makes counter-clockwise moment respective to the critical polygon. Thus, a toppling tip at height z_3 will initialize the toppling process. In case (b), v_3 is out of the circle and the interior angle at v_3 is less than 90 degree. The contact force at E passes exactly through one vertex of the critical polygon in gray and makes counter-clockwise moment respective to the critical polygon. Thus, a toppling tip at any height between E and v_3 will initialize the toppling process. In case (c), v_3 is within the circle and the interior angle at v_3 is greater 90 degree. Thus, no contact can topple the part and no solution exists.

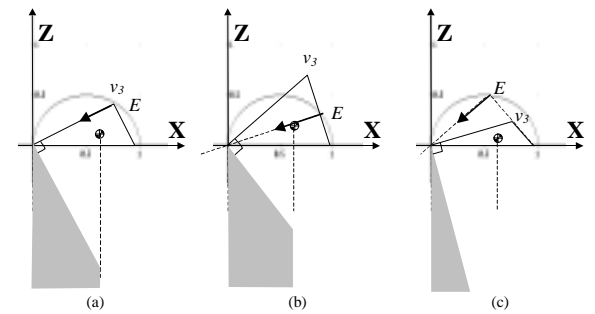


Figure 7 Geometric interpretation of the boundary.

6. DISCUSSION

We describe a new Java-based CAD tool that designs gripper jaws that align a given part from an initial

orientation to a desired final orientation. We describe algorithmic improvements in computational efficiency, frictional analysis and form closure analysis. We describe a Java-based graphical interface and give examples. We then apply the CAD tool to identify conditions under which jaw designs exist, leading to a formal characterization of a class of triangular parts for which no gripper design exists. Online CAD tools such as this can allow designers to rapidly check manufacturability and modify designs to avoid handling problems.

REFERENCES

- [1] T. Abell and M. Erdmann. "Stably supported rotations of a planar polygon with two frictionless Contacts," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, Pittsburgh, PA, 1995, pp. 411-418.
- [2] C. Anderson, Y. Zhuang, and K. Goldberg. "FixtureNet II: interactive redesign and force visualization on the web," in *CD Proc. ASME Design Eng. Tech. Conf.*, Sacramento, CA, 1997.
- [3] R.-P. Berretty, K. Goldberg, L. Cheung, M. Overmars, G. Smith, and A. F. van der Stappen. "Trap design for vibratory bowl feeders," in *Proc. IEEE Int. Con. Robot. Automat.*, Detroit, MI, 1999, pp. 2558-63.
- [4] R. Brost and K. Goldberg. "A complete algorithm for designing planar fixtures using modular components," *IEEE Trans. Robot. Automat.*, vol. 12, no. 1, pp. 31-46, 1999.
- [5] W. Chui and P. Wright, "A WWW computer integrated manufacturing environment for rapid prototyping and education," *J. Computer Integrated Manufacturing*, vol. 12, pp. 54-60, 1999.
- [6] J. Larson and H. Cheng. "Object-oriented cam design through the internet," *J. Intell. Manufacturing*, vol. 11, pp. 515-34, 2000.
- [7] K. Lynch. "Toppling manipulation," in *Proc. IEEE Int. Con. Robot. Automat.*, Detroit, MI, 1999, pp. 2551-7.
- [8] K. Lynch. "Inexpensive conveyor-based parts feeding," *Assembly Automat.*, vol. 19, no. 3, pp. 209-15, 1999.
- [9] M. T. Mason. "Two graphical methods for planar contact problems," in *Proc. IEEE/RSJ Int. Workshop Intell. Robots Syst.*, Osaka, Japan, 1991, pp. 443-8.
- [10] B. Mishra, J. Schwartz, and M. Sharir. "On the existence and synthesis of multifinger positive grips," *Algorithmica*, vol. 2, no. 4, pp. 541-58, 1987.
- [11] S. Piccinocchi, M. Ceccarelli, F. Piloni, and A. Bicchi. "Interactive benchmark for planning algorithms on the Web: <http://www.piaggio.cci.unipi.it/benchplanning.html>," in *Proc. IEEE Int. Con. Robot. Automat.*, Albuquerque, NM, 1997, pp.399-404.
- [12] F. Preparata and S. Hong. "Convex hulls of finite sets of points in two and three dimensions," *Communications of ACM*, vol. 20, no. 2, pp. 87-93, 1977.
- [13] A. Rao, D. Kriegman and K. Goldberg. "Complete algorithm for feeding polyhedral parts using pivot grasps," *IEEE Trans. Robot. Automat.*, vol. 12, no. 2, pp. 331-42, 1996.
- [14] C. Smith and P. Wright, "CyberCut: A World Wide Web based design to fabrication tool", *J. Manufacturing Syst.*, vol. 15, pp. 432-42, 1996.
- [15] G. Smith, E. Lee, K. Goldberg, K. Bohringer, and J. Craig. "Computing parallel-jaw grips," in *Proc. IEEE Int. Con. Robot. Automat.*, Detroit, MI, 1999, pp. 1897-903.
- [16] J. Trinkle and R. Paul. "Planning for dexterous manipulation with sliding contacts," *Int. J. Robot. Res.*, vol. 9, no. 3, pp. 24-48, 1990.
- [17] K. Van Katwyk and H. Cheng. "Xlinkage: a web-based analysis and simulation tool for planar mechanical systems," in *CD Proc. ASME Design Eng. Tech. Conf.*, Sacramento, CA, 1997.
- [18] R. Wagner, G. Castanotto, and K. Goldberg. "FixtureNet: interactive computer-aided design via the World Wide Web," *J. Human-Computer Studies*, vol. 46, pp. 773-88, 1997.
- [19] T. Zhang, G. Smith and K. Goldberg. "Compensatory grasping with the parallel-jaw gripper," in *Algorithmic and Computational Robotics: New Directions*, B. Donald, K. Lynch, and D. Rus, Eds., Natick, MA: A K Peters, 2001, pp. 169-79.
- [20] T. Zhang, G. Smith, R. Berretty, M. Overmars, and K. Goldberg. "The toppling graph: designing pin sequences for part feeding," in *Proc. IEEE Int. Con. Robot. Automat.*, San Francisco, CA, 2000, pp. 139-46.
- [21] T. Zhang and K. Goldberg. "Design of gripper jaws based on trapezoidal modules," in *Proc. IEEE Int. Con. Robot. Automat.*, Seoul, Korea, 2001.
- [22] T. Zhang. "Optimal Design of Robot Gripper Jaws," Ph.D. Dissertation, Dept. of Industrial Engineering and Operations Research, UC Berkeley, 2001.
- [23] <http://www.geom.umn.edu/apps>
- [24] <http://bart.aero.psu.edu/classes/ConvexHull/CGA>
- pp
- [25] <http://riot.ieor.berkeley.edu/>
- [26] <http://iel.ucdavis.edu/design/>
- [27] <http://cybercut.berkeley.edu/>